PreCalculus/ Honors

Summer Review Packet – 2018

DUE THE FIRST DAY OF CLASS

In preparation for the Fall Semester, this assignment is required to prepare you for your PreCalculus or PreCalculus Honors Course.

About PreCalculus:
Precalculus is the branch of mathematics pertaining to the study of prerequisites for the study of Calculus. It involves algebra, analytical geometry and trigonometry. A study of the concepts involved enables students to reason and problem solve effectively. Students use a graphing calculator as an integral tool in analyzing data and modeling functions to represent real world applications. Lacordaire Academy recommends a **TI-84 graphing calculator**. Students who purchase a different model will be responsible for its operation. It will be used throughout your high school and college career.

Expectations of the Summer Packet:
The problems in this packet are designed to help you review topics that are important to your success in Pre-Calculus. ***All work must be neatly shown for each problem***. The problems should be done correctly, not just attempted.

The packet is due the first day of school. During the first week(s) of school, concepts in the packet will be reviewed.

All work should be completed and ready to turn in on the first day of classes.

Some helpful sites:

www.purplemath.com/modules/index.html
www.coolmath.com
www.khanacademy.com
www.studentguide.org/a-complete-list-of-online-math-online-resources/
www.profrobbob.com (Tarrou's Chalk Talk)

Current students: For assistance in completing this packet, the electronic subscription associated with last year's math textbook is still available for your use until Aug 31st.

Enjoy your summer!

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WELCOME TO PRECALCULUS

What is PreCalculus? Pre-Calculus is divided into two major categories:

Trigonometry and Math Analysis

Trigonometry, which is the study of triangles, typically begins with an understanding of basic functions, then branches into how triangles and their angles can be drawn and represented in rotations, degrees and radian measure. With this foundation, students then introduced to the ***Unit Circle***, which enables them to explore trigonometric graphs, trigonometric identities, and trigonometric equations, as well as how to solve both right and *oblique* triangles.	**Math Analysis**, which some instructors sometimes call Algebra 3, digs deeper into algebra concepts. Specifically, functions, domain and range, and end behavior. The focus of math analysis is not to just review or solve more complicated equations, but to show students how to represent them in various formats (i.e., graphically, numerically, and verbally). In particular, students will study polynomial functions, rational functions, exponential and logarithmic functions and conic sections and learn how to express their findings using various modalities.

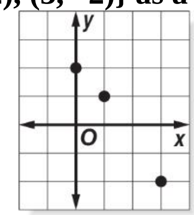
Contents

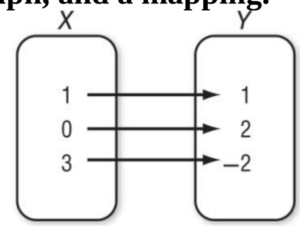
Section 1. Relations and Functions

A **relation** is a set of ordered pairs. A relation can be represented by a set of ordered pairs, a table, a graph, or a **mapping**. A mapping illustrates how each element of the domain is paired with an element in the range. The set of first numbers of the ordered pairs is the **domain**. The set of second numbers of the ordered pairs is the **range** of the relation.

Example: a. Express the relation {(1, 1), (0, 2), (3, –2)} as a table, a graph, and a mapping.

x	y
1	1
0	2
3	– 2





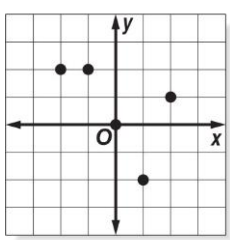
b. Determine the domain and the range of the relation.
The domain for this relation is {0, 1, 3}. The range for this relation is {–2, 1, 2}.

Identify Functions
Relations in which each element of the domain is paired with exactly one element of the range are called **functions**.

1. What is the vertical line test?

Determine whether each relation is a function

2.



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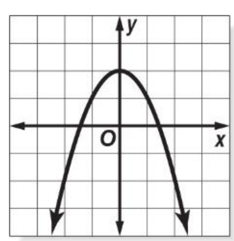

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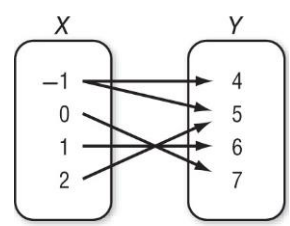


5. {(–3, –3), (–3, 4), (–2, 4)}

6. $-2x + 4y = 0$

7. $x^2 + y^2 = 8$

Exercises
If $f(x) = 2x - 4$ and $g(x) = x^2 - 4x$, find each value.

8. $f(4)$

9. $g(2)$

Section 2. Solving Quadratic Equations by Factoring

One of the ***most important*** skills you will need is the ability to factor expressions, and apply those concepts to more complex expressions.

Solve Equations by Factoring When you use factoring to solve a quadratic equation, you use the following property.

Zero Product Property	For any real numbers *a* and *b*, if *ab* = 0, then either *a* = 0 or *b* =0, or both *a* and *b* = 0.

Example: Solve each equation by factoring.

a. $3x^2 = 15x$		**b.** $4x^2 - 5x = 21$	
$3x^2 = 15x$	Original equation	$4x^2 - 5x = 21$	Original equation
$3x^2 - 15x = 0$	Subtract 15*x* from both sides.	$4x^2 - 5x - 21 = 0$	Subtract 21 from both sides.
$3x(x - 5) = 0$	Factor the binomial.	$(4x + 7)(x - 3) = 0$	Factor the trinomial.
$3x = 0$ or $x - 5 = 0$	Zero Product Property	$4x + 7 = 0$ or $x - 3 = 0$	Zero Product Property
$x = 0$ or $x = 5$	Solve each equation.	$x = -\frac{7}{4}$ or $x = 3$	Solve each equation.
The solution set is {0, 5}.		The solution set is $\left\{-\frac{7}{4}, 3\right\}$	

Solve the quadratic equations by factoring.

10. $x^2 - x - 6$

11. $x^2 - 4x - 21$

12. $x^2 - 22x + 121$

Section 3. Solving Quadratic Equations by Completing the Square

Complete the Square To complete the square for a quadratic expression of the form $x^2 + bx$, follow these steps.

1. Find $\frac{b}{2}$. ⟶ **2.** Square $\frac{b}{2}$. ⟶ **3.** Add $\left(\frac{b}{2}\right)^2$ to $x^2 + bx$.

Example 1	Example 2
Example 1: Find the value of *c* that makes $x^2 + 22x + c$ a perfect square trinomial. Then write the trinomial as the square of a binomial. **Step 1** $b = 22; \frac{b}{2} = 11$ **Step 2** $11^2 = 121$ **Step 3** $c = 121$ The trinomial is $x^2 + 22x + 121$, which can be written as $(x + 11)^2$.	**Example 2: Solve $2x^2 - 8x - 24 = 0$ by completing the square.** $2x^2 - 8x - 24 = 0$ Original equation $\frac{2x^2 - 8x - 24}{2} = \frac{0}{2}$ Divide each side by 2. $x^2 - 4x - 12 = 0$ $x^2 - 4x - 12$ is not a perfect square. $x^2 - 4x = 12$ Add 12 to each side. $x^2 - 4x + 4 = 12 + 4$ Since $\left(\frac{4}{2}\right)^2 = 4$, add 4 to each side. $(x - 2)^2 = 16$ Factor the square. $x - 2 = \pm 4$ Square Root Property $x = 6$ or $x = -2$ Solve each equation. The solution set is $\{6, -2\}$.

13. $x^2 - 4x + 3 = 0$

14. $x^2 + 10x = -9$

15. $x^2 - 8x - 9 = 0$

Section 4. Solving Quadratic Equations by the Quadratic Formula

You may encounter equations that look like they are in "quadratic form". You need to be able to recognize these quadratic-like and use the same principles to solve them.

Roots and the Discriminant of a quadratic equation in the form $ax^2 + bx + c = 0$

Discriminant	The expression under the radical sign, $b^2 - 4ac$, in the Quadratic Formula is called the **discriminant**.

Discriminant	Type and Number of Roots
$b^2 - 4ac > 0$ and a perfect square	2 rational roots
$b^2 - 4ac > 0$, but **not** a perfect square	2 irrational roots
$b^2 - 4ac = 0$	1 rational root
$b^2 - 4ac < 0$	2 complex roots

Example: Find the value of the discriminant for each equation. Then describe the number and type of roots for the equation.

a. $2x^2 + 5x + 3 = 0$ The discriminant is $b^2 - 4ac = 5^2 - 4(2)(3)$ or 1. The discriminant is a perfect square, so the equation has 2 rational roots.	**b.** $3x^2 - 2x + 5 = 0$ The discriminant is $b^2 - 4ac = (-2)^2 - 4(3)(5)$ or –56. The discriminant is negative, so the equation has 2 complex roots.

Quadratic Formula The **Quadratic Formula** can be used to solve *any* quadratic equation once it is written in the form $ax^2 + bx + c = 0$.

Quadratic Formula	The solutions of $ax^2 + bx + c = 0$, with $a \neq 0$, are given by $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Example: Solve $x^2 - 5x = 14$ by using the Quadratic Formula.

Rewrite the equation as $x^2 - 5x - 14 = 0$.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Quadratic Formula

$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-14)}}{2(1)}$ Replace *a* with 1, *b* with –5, and *c* with –14.

$= \frac{5 \pm \sqrt{81}}{2}$ Simplify.

$= \frac{5 \pm 9}{2}$

$= 7$ or -2

The solutions are –2 and 7.

State the value of the discriminant for each equation and the type of solutions. Then determine the solutions of the equation.

16. $3x^2 + 2x - 3 = 0$

17. $3x^2 - 7x - 8 = 0$

18. $2x^2 - 10x - 9 = 0$

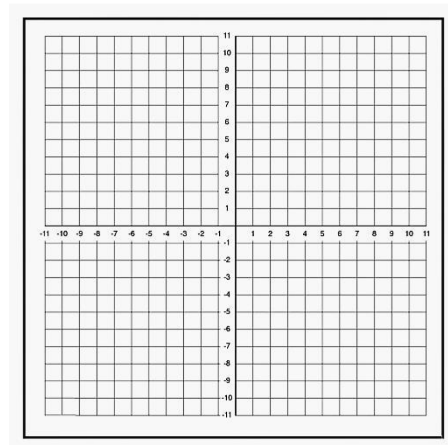
Section 5. Solving Polynomial Functions

19. Solve the following polynomial $x^4 - 6x^3 - 21x^2 + 34x + 48 = 0$

a. By the Fundamental Theorem of Algebra, how many solutions are we looking for? ______________

b. Using the Rational Zero Theorem, determine all the possible rational zeros.

c. Create the table of values and sketch the graph.



d. Describe the end behavior of the polynomial using limits

e. Using Descartes Rule of Signs, determine how many possible real positive and negative zeros there could be.

possible positive real zeros ________________

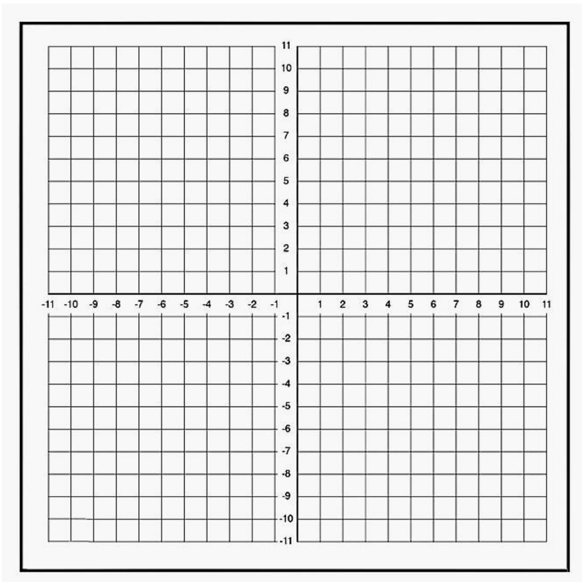
possible negative real zeros ________________

f. Determine all of the solutions (real and complex) of f(x).

Section 6. Solving Systems of Equations

Solve the system of equations using all 5 methods

20. Solve the system by graphing $y = -3x + 1$ **and** $y = 3x + 1$



21. Solve the system using substitution	**22. Solve the system using elimination**
$5x - y = 7$ $7x - y = 11$	$2x - y = 4$ $7x + 3y = 27$

23. Solve the system using Cramer's Rule	24. Solve the system using an inverse matrix
$8x + 3y = -7$ $7x + 2y = -3$	$2x + 5y = 11$ $4x + 3y = 1$

Section 7. Solving Exponential and Logarithmic Equations

Solve Exponential Equations All the properties of rational exponents that you know also apply to real exponents. Remember that $a^m \cdot a^n = a^{m+n}$, $(a^m)^n = a^{mn}$, and $a^m \div a^n = a^{m-n}$.

Property of Equality for Exponential Functions	If b is a positive number other than 1, then $b^x = b^y$ if and only if $x = y$.

Solving Logarithmic Equations

Property of Equality for Logarithmic Functions	If b is a positive number other than 1, then $\log_b x = \log_b y$ if and only if $x = y$.

Example 1: Solve $4^{x-1} = 2^{x+5}$.

$4^{x-1} = 2^{x+5}$	Original equation
$(2^2)^{x-1} = 2^{x+5}$	Rewrite 4 as 2^2.
$2(x-1) = x + 5$	Prop. of Equality for Exponential Functions
$2x - 2 = x + 5$	Distributive Property
$x = 7$	Subtract x and add 2 to each side.

Example 2: Write an exponential function for the graph that passes through the points (0, 3) and (4, 81).

Find the values of a and b to substitute into the general form of the exponential function $y = ab^x$. The y-intercept is (0, 3), so $a = 3$. Since the other point is (4, 81), by substitution $81 = 3b^4$. Solving for b yields $b = \sqrt[4]{27}$ or about 2.280. So, the equation is $y = 3(2.280)^x$.

Example 1: Solve $\log_2 2x = 3$.

$\log_2 2x = 3$	Original equation
$2x = 2^3$	Definition of logarithm
$2x = 8$	Simplify.
$x = 4$	Simplify.

The solution is $x = 4$.

Example 2: Solve the equation $\log_2 (x + 17) = \log_2 (3x + 23)$.

Since the bases of the logarithms are equal, $(x + 17)$ must equal $(3x + 23)$.

$(x + 17) = (3x + 23)$

$-6 = 2x$

$x = -3$

Exercises

Solve each exponential equation.

25. $9^{x+1} = 27^{x+4}$

26. $36^{2x+4} = 216^{x+5}$

27. $\left(\frac{1}{64}\right)^{x-2} = 16^{3x+1}$

Write an exponential function for the graph that passes through the given points.

28. (0, 4) and (2, 36)

29. (0, 6) and (1, 81)

Solve each logarithmic equation.

30. $\log_4 (5x + 1) = 2$

31. $\log_8 (x - 5) = \frac{2}{3}$

32. $\log_4 (3x - 1) = \log_4 (2x + 3)$

Section 8. Solving Exponential and Logarithmic Inequalities

Solve Exponential Inequalities An **exponential inequality** is an inequality involving exponential functions.

Property of Inequality for Exponential Functions	If $b > 1$, then $b^x > b^y$ if and only if $x > y$ and $b^x < b^y$ if and only if $x < y$.

Example: Solve $5^{2x-1} > \frac{1}{125}$.

$5^{2x-1} > \frac{1}{125}$	Original inequality
$5^{2x-1} > 5^{-3}$	Rewrite $\frac{1}{125}$ as 5^{-3}.
$2x - 1 > -3$	Prop. of Inequality for Exponential Functions
$2x > -2$	Add 1 to each side.
$x > -1$	Divide each side by 2.

The solution set is $\{x \mid x > -1\}$.

Solving Logarithmic Inequalities

Property of Inequality for Logarithmic Functions	If $b > 1$, $x > 0$, and $\log_b x > y$, then $x > b^y$. If $b > 1$, $x > 0$, and $\log_b x < y$, then $0 < x < b^y$. If $b > 1$, then $\log_b x > \log_b y$ if and only if $x > y$, and $\log_b x < \log_b y$ if and only if $x < y$.

Example 1: **Solve $\log_5 (4x - 3) < 3$.**

$\log_5 (4x - 3) < 3$ Original equation

$0 < 4x - 3 < 5^3$ Property of Inequality

$3 < 4x < 125 + 3$ Simplify.

$\frac{3}{4} < x < 32$ Simplify.

The solution set is $\left\{x \middle| \frac{3}{4} < x < 32\right\}$.

Example 2: **Solve the inequality $\log_3 (3x - 4) < \log_3 (x + 1)$.**

Since the base of the logarithms are equal to or greater than 1, $3x - 4 < x + 1$.

$2x < 5$

$x < \frac{5}{2}$

Since $3x - 4$ and $x + 1$ must both be positive numbers, solve $3x - 4 = 0$ for the lower bound of the inequality.

The solution is $\left\{x \middle| \frac{4}{3} < x < \frac{5}{2}\right\}$.

Solve each exponential inequality.

33. $7^{3x} < 49^{1-x}$

34. $8^{2x-5} < 4^{x+8}$

Solve each logarithmic inequality.

35. $\log_3 (x + 3) < 3$

36. $\log_{27} 6x > \frac{2}{3}$

Section 9. Solve Rational Equations

A **rational equation** contains one or more rational expressions. To solve a rational equation, first multiply each side by the least common denominator of all of the denominators. Be sure to exclude
any solution that would produce a denominator of zero.

Example: Solve $\frac{9}{10} + \frac{2}{x+1} = \frac{2}{5}$. Check your solution.

$\frac{9}{10} + \frac{2}{x+1} = \frac{2}{5}$	Original equation
$10(x+1)\left(\frac{9}{10} + \frac{2}{x+1}\right) = 10(x+1)\left(\frac{2}{5}\right)$	Multiply each side by 10(*x* + 1).
$9(x+1) + 2(10) = 4(x+1)$	Multiply.
$9x + 9 + 20 = 4x + 4$	Distribute.
$5x = -25$	Subtract 4*x* and 29 from each side.
$x = -5$	Divide each side by 5.

Check $\frac{9}{10} + \frac{2}{x+1} = \frac{2}{5}$	Original equation
$\frac{9}{10} + \frac{2}{-5+1} \stackrel{?}{=} \frac{2}{5}$	$x = -5$
$\frac{18}{20} - \frac{10}{20} \stackrel{?}{=} \frac{2}{5}$	Simplify.
$\frac{2}{5} = \frac{2}{5}$	

Solve each equation. Check your solution. State where the equation is undefined.

37. $\frac{4}{x-1} = \frac{x+1}{12}$

38. $\frac{x}{x-2} + \frac{4}{x-2} = 10$

Section 10. Solving Rational Inequalities

To solve a rational inequality, complete the following steps.

Step 1	State the excluded values.
Step 2	Solve the related equation.
Step 3	Use the values from steps 1 and 2 to divide the number line into regions. Test a value in each region to see which regions satisfy the original inequality.

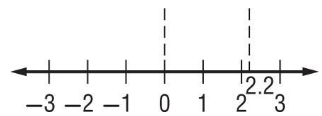
Example: Solve $\frac{2}{3n} + \frac{4}{5n} \leq \frac{2}{3}$.

Step 1 The value of 0 is excluded since this value would result in a denominator of 0.

Step 2 Solve the related equation.

$\frac{2}{3n} + \frac{4}{5n} = \frac{2}{3}$	Related equation
$15n\left(\frac{2}{3n} + \frac{4}{5n}\right) = 15n\left(\frac{2}{3}\right)$	Multiply each side by 15n.
$10 + 12 = 10n$	Simplify.
$22 = 10n$	Add.
$2.2 = n$	Divide each side by 10.

Step 3 Draw a number with vertical lines at the excluded value and the solution to the equation.



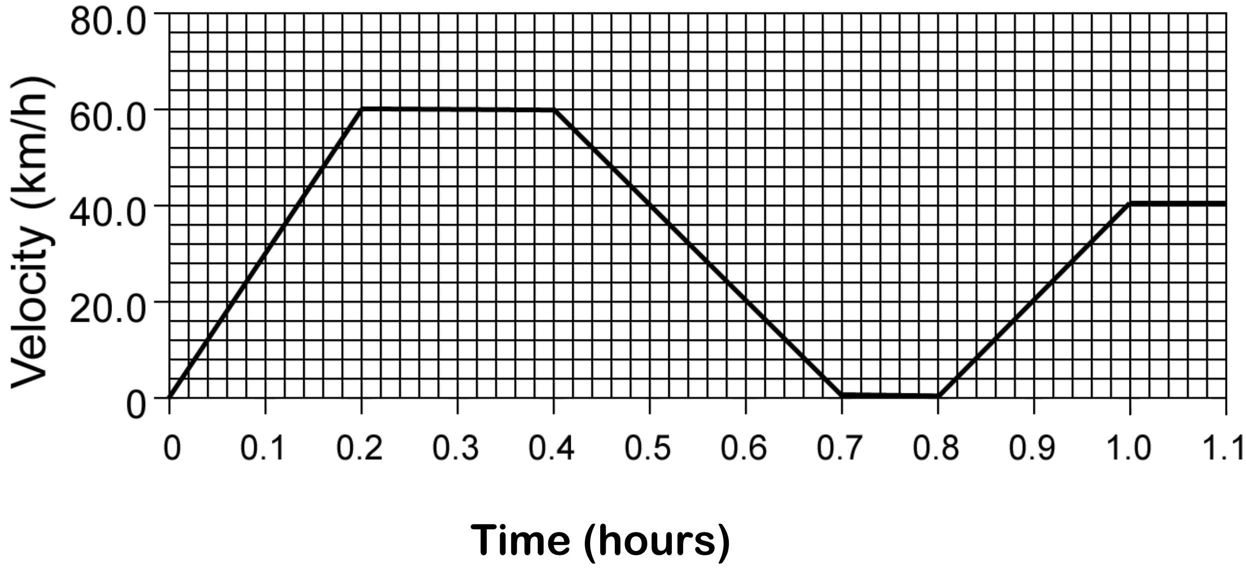
Solve each inequality. Check your solutions.

39. $\frac{3}{2x} - \frac{2}{x} > \frac{1}{4}$

40. $\frac{4}{x-1} + \frac{5}{x} < 2$

Section 11. Interpreting Graphs

Velocity-Time Graph



41. Above is a velocity-time graph of a moving car. Answer the following questions using the graph.

___________ a. At what time was the car stopped?

___________ b. At what time did the car have the greatest velocity?

___________ c. What was the greatest velocity?

___________ d. At what time(s) was the car accelerating?

___________ e. How fast was the car going at 1.0 h?

___________ f. What is the acceleration at 0.9 hr?